REVIEW

THE NEW INDEX NUMBERS OF THE UNITED STATES BUREAU OF LABOR STATISTICS

THE United States Bureau of Labor Statistics has adopted a new method of computing index numbers and the change in method has been made the occasion for a very full and thoro discussion of the subject. This discussion is to be found in Bulletins No. 173 and No. 181 and Dr. Wesley C. Mitchell is to be congratulated on the thoroness and lucidity of his exposition. The former includes a brief history of index numbers, a statement of the paramount importance of obtaining accurate statistics from which index numbers may be computed, together with the difficulties of collecting such data, a carefully balanced comparison of the relative merits and demerits of different types of averages and aggregates, and concludes with an account of the tables of index numbers prepared by other statisticians in the United States and elsewhere. The matter in Bulletin No. 181, is devoted to an explanation of the new methods employed in computing index numbers and the reasons for making the change.

Among the interesting topics discussed in the two papers are the relative merits of chain index numbers and index numbers computed to a fixed base, and methods of substituting, adding new items, or dropping old ones, without breaking the continuity of the series as, in a dynamic society, changes in tastes and fashions and in the commodities consumed take place from year to year.

Especially is the importance of gathering and publishing in full the price data emphasized. Indeed this is accorded an importance superior to the particular type of average or

aggregate which may be adopted in manipulating the data. In this connection Professor Taussig's words are especially pertinent: "In regard to all these suggestions, whether for improvement in the arithmetic mean or for the use of a different mean, it must be borne in mind that no index number corresponds to a real thing. It is not like the mean of certain observations in natural science — such, for example, as those for measuring the distance between the earth and the sun — of which any one may err, but whose average will point to a single specific fact. An index number points to no single fact."[1] This being true, it follows that there is no single best method of computing index numbers. Both the items selected and the type of average employed have reference to the purpose in view. What may most adequately represent the facts in one case may be misleading in another. The problem, then, for a compiler of general index numbers is to establish general tendencies in regard to prices most likely to represent the case without serious error for whatever purpose the tables may be used.

When the end in view is specific and capable of precise statement the problem of choosing methods is comparatively simple. Straightforward logic then determines what commodities should be included, what sources of quotations should be drawn upon, and how the original data should be worked up to give the most significant results. Puzzles a-plenty are left, but most of them are limited to finding the best compromise between what logic marks out as desirable and what is feasible in view of the time and money at the investigator's disposal.

Few of the widely-used index numbers, however, are made to serve one special purpose. On the contrary, most of them are "general-purpose" series, designed with no aim more definite than that of measuring changes in the price level. Once published they are used for many ends — to show the depreciation of gold, the rise in the cost of living, the alternations of business prosperity and depression, and the allowance to be made for changed prices in comparing estimates of national wealth or private income at different times. . . .

The compiler of a general-purpose index number, then, cannot foresee to what uses and misuses his figures will be put. For each of

[1] Taussig, Principles of Economics, vol. i, p 294.

the legitimate uses he might conceivably devise an appropriate series. But he cannot conceivably devise a single series that will serve all uses equally well. For the very qualities that make an index number good, say, for the man of affairs concerned with the business outlook, may make it bad for other men interested in the fortunes of farmers, in the effects of the tariff, in the relation between gold output and prices, in comparing changes in price levels in different countries, etc. The day has not yet come when the uses of index numbers are sufficiently differentiated and standardized to secure the regular publication of numerous special-purpose series. Until that day does come the making of general-purpose series will continue, and the makers will go on choosing their methods perforce on rather vague and general grounds. So long also must most users of index numbers put up with figures imperfectly adapted to their ends.[1]

The above quotation sufficiently justifies Dr. Mitchell in emphasizing the importance of publishing copious data in full so that the special investigator may have the opportunity of making a selection of material and method most suited to his purpose. The change in the method of computing index numbers recently made by the Bureau is an attempt at a better solution of the general-purpose index-number problem to which Dr. Mitchell refers.

In previous issues of the Bulletin, index numbers were computed by the method of simple averages of relatives. Taking the average of the prices of each of the commodities included in the table from 1890 to 1899 inclusive as a base (100) the relative price for any given year has then been found and a simple average (arithmetic mean) of these relatives has been taken as the index number for that year. The new method is based not upon a simple average of relatives, but upon aggregates of the total value in exchange of each of the commodities. Each commodity is therefore weighted in proportion to its importance in commerce. Moreover the base is changed from an average of prices for the period 1890–1899 to the aggregate of values for the single year 1914. A simple illustration will explain the difference better than words. Suppose we have given the following data:

[1] Bulletin of the United States Bureau of Labor Statistics, No 173, pp 25–26.

Commodity	Unit	Price			Quantity Exchanged in 1909 000,000 Omitted
		Average 1890–1899	1910	1914	
Cotton	Pound	$0.08	$0.15	$0.12	3,500
Hay.......	Ton	10.40	17.25	15.70	11
Wheat.....	Bushel	.75	1.10	1.04	27

OLD METHOD

Commodity	Price 1890–1899	Relative	Price 1910	Relative
Cotton..........	$0.08	100	$0.15	187.5
Hay.............	10.40	100	17.25	165.9
Wheat...........	.75	100	1.10	146.6
	——	3)300	——	3)500.0
		100		166.7

Index number for 1910, 166.7.

NEW METHOD

Commodity	Total Exchange Value in	
	1914	1910
Cotton.........	(3,500 × .12) $420.00	(3,500 × .15) $525.00
Hay............	(11 × 15.7) 172.70	(11 × 17.25) 189.75
Wheat..........	(27 × 1.04) 28.08	(27 × 1.10) 29.70
	$620.78	$744.45

000,000 omitted in above aggregate.
620.78 : 744.45 = 100 : 119.9.
Index number for 1910, 119.9.

As the above index numbers are computed to a different base they cannot be compared. For purposes of comparison, however, tables are published (Bulletin No. 181, pp. 12–16) in which, in parallel columns, index numbers to each base are reduced to terms of the other.

In what respects is the new system to be preferred to the old? The advantage of the weighted average in that it makes allowance for the difference in importance of commodities, and the objection to the arithmetic mean, that it gives to extreme variations in the prices of a few commodities undue influence in determining the index number, have been generally recognized. It is this objection that has led some statisticians to prefer the geometric mean or the median. The new method of the Bureau, however, is open to the same objection on this score as the arithmetic mean. Since the weighting for each commodity (the quantity exchanged in 1909) is constant throughout the whole series of index numbers, extreme variations in price multiplied by constant multipliers affect the aggregates in much the same way that the same variations would affect the relatives. Indeed, if the commodity affected with the extreme variation be also one having a very large volume of sales the effect on the index number will be even greater. But perhaps the very fact of the weighting justifies the effect on the index number. If a commodity which is very much in demand rises sharply in price, then even tho it be only a single commodity people really are forced to pay more for their usual satisfactions and the index number should show this fact.

It was noted in the last paragraph that the weighting for each commodity remains constant throughout the series. As the theory of the compiler is that the most satisfactory table of index numbers for general use is one which will show how much a typical person will be forced to pay for "the same bill of goods" from year to year, this could not well be otherwise. But it raises a question. For how long a period is the same weighting to be retained? And when the weighting is changed, how is the series of index numbers based upon one weighting to be adjusted to the series based upon another? Clearly, with new inventions constantly springing up, and with frequent changes in tastes and fashions, commodities change in their relative importance as well as in their prices. Sooner or later some method of "splicing" will become imperative. The difficulty is by no means insuperable;

indeed, methods of "splicing" when new commodities are added or old ones dropped are discussed in the paper, but this particular problem is somewhat different and will ultimately have to be faced.

One advantage of the new system is that the series can, if desired, be quickly changed to a new base. The index numbers in former issues of the Bulletin were computed to a fixed base, namely, the average of prices from 1890 to 1899. But we do not so often care to know how prices today compare with prices in the 90's as how they compare with the prices of three or four years ago, and in what direction and to what extent they are changing now. With the new tables, whatever be the base, the index numbers can be easily and accurately adjusted to any other base by simply dividing each of the index numbers (multiplied by 100) by the index number corresponding with the new base. For example, the index numbers for the decade 1900–09 to the base 1914 (100) are

1900	1901	1902	1903	1904	1905	1906	1907	1908	1909
81	80	86	86	87	86	89	95	92	98

If it is desired to reduce these index numbers to the base 1900 (100) we should multiply each by 100 and divide the products by 81,

1900	1901	1902	1903	1904	1905	1906	1907	1908	1909
100	99	106	106	107	106	110	117	114	121

But this process applied to a series of index numbers computed from a simple average of relatives does not lead to accurate results. In strict accuracy it is necessary to recompute all the relatives separately to the new base.[1]

[1] *Old method (average of relatives)*

Let n equal the number of commodities and p_1, p_2, p_3, etc., be their prices in any given year, say, 1910, and P_1, P_2, P_3, etc., be the prices for the same commodities in the base year, say, 1890. Then (1) $\frac{100}{n}\left[\frac{p_1}{P_1}+\frac{p_2}{P_2}+\ldots\right]$ will be the index numbers for 1910. By the same reasoning, when π_1, π_2, π_3, etc., are the prices in 1912, (2) $\frac{100}{n}\left[\frac{\pi_1}{P_1}+\frac{\pi_2}{P_2}+\ldots\right]$ will be the index number for 1912. Again, by the same reasoning, if it is desired to compute the index number for 1912 to the base 1910, we

One interesting point made by Dr. Mitchell deserves notice in this connection. It is claimed that a series of index numbers computed to a fixed base becomes progressively untrustworthy in proportion as the dates for which the index numbers are computed become more and more remote from the date of the base, and that for this reason chain-index numbers, that is, index numbers in which each year is successively taken as a base in computing the index number for the next year, are more reliable. The reason for this is that if we compare prices on successive years the great mass of the prices will exhibit only a slight change, there will be an obvious central tendency up or down with comparatively few extreme variations either of increase or decrease; whereas, if we compare prices separated by a long interval of time the extreme variations become more and more numerous and the central tendency less decisive. This point is developed with considerable mathematical detail through the use of decils and normal probability curves and the contention is undoubtedly made good that prices at a remote interval of time from the fixed base show a much greater scattering than those in adjacent years. But are the index numbers derived from them therefore more untrustworthy? If in the course of twenty years the price of wheat has advanced thirty per cent and the price of linen has fallen twenty per cent is the index number showing a rise of prices any less to be relied on than

shall have (3) $\frac{100}{n}\left[\frac{\pi_1}{p_1}+\frac{\pi_2}{p_2}+\ .\ \right]$ But a moment's inspection of the above formulae shows that (3) cannot be obtained by dividing (2), multiplied by 100, by (1), as would be the case if the reduction could be made as above.

New Method (Aggregates)

Using the same symbols, with the addition that, q_1, q_2, q_3, etc, represent the quantities exchanged, we have

(1) $\frac{100\ (q_1 p_1 + q_2 p_2 + \ .\ \ )}{q_1 P_1 + q_2 P_2 + \ \ .}$ = index number for 1910, base 1890 (100)

(2) $\frac{100\ (q_1 \pi_1 + q_2 \pi_2 + \ \ \ )}{q_1 P_1 + q_2 P_2 + \ .\ .}$ = index number for 1912, base 1890 (100)

(3) $\frac{100\ (q_1 \pi_1 + q_2 \pi_2 \ \ \ )}{q_1 p_1 + q_2 p_2 + \ \ .}$ = index number for 1912, base 1910 (100)

But (3) is obviously equal to (2), multiplied by 100, ÷ (1).

would be an index number derived from a three per cent rise and two per cent fall of the corresponding commodities in adjacent years? If so, we are confronted with a curious paradox. If the mere fact that prices tend to become scattering with the lapse of time tends to cast suspicion on index numbers irrespective of the method of computation, then this defect should attach to the method of aggregates as well as to the method of relatives. For if prices become scattering after the lapse of time so also will weighted prices. Now suppose that taking 1890 as a base we compute by the new method index numbers for 1910 and 1911. By the reasoning just given these index numbers should be untrustworthy. But as was shown on page 802 we can accurately compute the 1911 index number to the 1910 base by the simple division of the 1911 index number (multiplied by 100) to the 1890 base by the index number for 1910 to the same base. That is, by dividing one unreliable index number by another we get a reliable index number!

It has been impossible in the space of this review to cover more than a few of the points touched upon by Dr. Mitchell in his admirable paper. It deserves to be carefully read by all students of the problems of index numbers and by all persons who have occasion to use them. As touching the merits of the new tables[1] it may be well to sum up the advantages claimed for them in Dr. Mitchell's own words.

> The technical difficulties attending the construction of index numbers made of actual prices, then, can be surmounted. Offsetting these difficulties are numerous and substantial advantages. Aggregates of money prices weighted according to the importance of the several articles are as easy to understand as arithmetic means of relative prices. They are less laborious to compute than any other form of weighted series, for no relative prices are used; the original quotations are multiplied directly by the physical quantities used as weights, and the products added together. They are not tied to a single base period; but from them relative prices can quickly be made upon the chain system or any fixed base that is desired, and

[1] The aggregates, however, are not published. The tables include index numbers derived from the aggregates, and prices and weightings from which the aggregates may be derived.

these relative prices themselves can be shifted about at will as readily as geometric means. Hence they are capable of giving direct comparisons between prices on any two dates in which the investigator happens to be interested. Hence, also, they can be compared with any index numbers covering the same years, on whatever base the latter are computed. Their meaning is perfectly definite — which is not always true of medians. They cannot be made to give apparently inconsistent results like arithmetic means. When published as sums of money, they can be added, subtracted, multiplied, divided, or averaged in any way that is convenient. When weighted on a sound system, they cannot be unduly distorted by a very great advance in the price of a few articles, and yet, unlike medians, they allow every change in the price of every article to influence the result. . . .

In addition to the advantages peculiar to themselves, aggregates of actual prices can readily be given all the advantages claimed for weighted arithmetic means of relative prices. This combination of qualities makes them the most desirable type of general-purpose index numbers.[1]

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[1] Bulletin of the United States Bureau of Labor Statistics, pp 91, 93.